Oscillatory Screening Effects on Elastic Collisions in Dense Electron-Ion Quantum Plasmas

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Z. Naturforsch. **64a,** 237 – 241 (2009); received April 4, 2008 / revised September 15, 2008

The oscillatory screening effects on elastic electron-ion collisions are investigated in dense quantum plasmas. The eikonal method with the modified Debye-Hückel potential is employed to obtain the scattering phase shift and scattering amplitude. In addition, the total elastic collision cross section is obtained by the optical theorem with the forward scattering amplitude in quantum plasmas. It is shown that the modified Debye-Hückel screening in quantum plasmas produces the oscillatory behaviour of the scattering phase shift. In addition, the minimum position of the phase shift is receded from the target ion with decreasing the quantum wave number. It is also found that the oscillatory screening effect suppresses the differential cross section. The total cross section is also found to be decreased due to the oscillatory screening effect. In addition, it is shown that the total cross section decreases with an increase of the quantum wave number.

Key words: Oscillatory Screening; Quantum Plasmas.

The electron-ion collision [1, 2] has received much attention since this is one of the most fundamental processes and also has wide applications in many areas of physics. Recently, atomic collision processes in plasmas have been extensively investigated as useful tools for plasma diagnostics. It has been shown that the elastic electron-ion collision plays an important role in electric conductivity [3] in various plasmas. The screened interaction between charged particles in classical plasmas has been mainly described by the Yukawa-type standard Debye-Hückel potential [1]. Furthermore, it would be anticipated that the multiparticle correlation effects caused by simultaneous interaction of many particles in plasmas should be taken into account to describe the interaction potential with an increase of the plasma density. Recently, there has been a great interest in investigating physical properties of various quantum plasmas [4, 5]. Moreover, the quantum plasmas have been found in many nano-scale objects such as nano-wires, quantum dot, semiconductor devices as well as in laser-produced dense plasmas and also in astrophysical compact objects [6, 7]. Very recently, the modified Debye-Hückel potential including the oscillatory behaviour in dense electron-ion quantum plasmas has been obtained by using the linear dielectric response formalism [8]. Hence, it is expected that the electron-ion collisions in quantum plasmas would be quite different from those in classical plasmas. Thus, in the present paper we investigate the oscillatory screening effects on elastic collisions in dense electron-ion quantum plasmas. The eikonal analysis with the modified Debye-Hückel potential is employed to obtain the scattering phase shift and scattering amplitude. In addition, the differential and total elastic collision cross sections are obtained by the optical theorem with the forward scattering amplitude in quantum plasmas.

In the presence of the potential field, the wave function $\Psi_k(\mathbf{r})$ can be represented by the following form of the Schrödinger equation:

$$(\nabla^2 + k^2)\Psi_k(\mathbf{r}) = U(\mathbf{r})\Psi_k(\mathbf{r}), \tag{1}$$

where k $[= (2\mu E/\hbar^2)^{1/2}]$ is the wave number, μ is the reduced mass of the collision system, E $(= \mu \upsilon^2/2)$ is the collision energy, υ is the collision velocity, $\hbar$ is the rationalized Planck constant, $\mathbf{r}$ is the position vector, $U(\mathbf{r})$ $[\equiv (2\mu/\hbar^2)V(\mathbf{r})]$ is the reduced potential, and $V(\mathbf{r})$ is the interaction potential. It is essential to investigate the scattering amplitude in order to obtain the physical properties of the collision system since the scattering process would be reduced to the problem of finding the scattering amplitude using various techniques. Using the eikonal method for the potential scattering with the free outgoing Green's function of the Helmholtz operator $(\nabla^2 + k^2)$, $G_0^{(+)}(\mathbf{r},\mathbf{r}')$ $(= -\mathrm{e}^{\mathrm{i}k|\mathbf{r}-\mathbf{r}'|}/4\pi|\mathbf{r}-\mathbf{r}'|)$ [9], and the wave function $\Psi_k(\mathbf{r})$ in the cylindrical coordinate system such as $\mathbf{r} = \mathbf{b} + z\hat{\mathbf{n}}$, where $\mathbf{b}$ is the impact parameter, $\hat{\mathbf{n}}$ is the unit vector perpendicular to the momentum transfer $\mathbf{K}$ $(\equiv \mathbf{k}_\mathrm{f} - \mathbf{k}_\mathrm{i})$, $\mathbf{k}_\mathrm{i}$ and $\mathbf{k}_\mathrm{f}$ are, respectively, the incident and final wave vectors, the eikonal scattering amplitude [9] is given by

$$f_E(\mathbf{k}_\mathrm{f},\mathbf{k}_\mathrm{i}) = -\mathrm{i}k\int_0^\infty \mathrm{d}b\, b\, J_0(Kb)\,\{\exp[\mathrm{i}\xi(k,b)] - 1\}, \tag{2}$$

where $k \equiv k_\mathrm{f} = k_\mathrm{i}$ for the elastic collision, $K = 2k\sin(\theta/2)$, θ is the scattering angle between $\mathbf{k}_\mathrm{f}$ and $\mathbf{k}_\mathrm{i}$, $J_0(Kb)$ is the zeroth-order first kind Bessel function [10], and $\xi(k,b)$ is the scattering phase shift [9]:

$$\xi(k,b) = -\frac{1}{2k}\int_{-\infty}^{\infty} \mathrm{d}z\, U(b,z). \tag{3}$$

It is known that the optical theorem [11] provides the general relation between the total cross section of all collision processes and the imaginary part of the forward scattering amplitude. Hence, according to the optical theorem [11], the total elastic collision cross section σ_el is represented by

$$\begin{aligned}\sigma_\mathrm{el}(k) &= \frac{4\pi}{k}\mathrm{Im} f_E(\theta = 0)\\ &= 4\pi\int_0^\infty \mathrm{d}b\, b\{1 - \cos[\xi(k,b)]\},\end{aligned} \tag{4}$$

where Im stands for the imaginary part, and $f_E(\theta = 0)$ is the forward scattering amplitude, i. e., if $K = 0$:

$$f_E(\theta = 0) = -\mathrm{i}k\int_0^\infty \mathrm{d}b\, b\{\exp[\mathrm{i}\xi(k,b)] - 1\}. \tag{5}$$

Very recently, an excellent work of Shukla and Eliasson [8] has provided the remarkably useful form of the modified Debye-Hückel potential of a test charge in dense electron-ion quantum plasmas by using the linear dielectric response formalism. Using this effective screening model [8], the modified Debye-Hückel interaction potential $V_\mathrm{MDH}(r)$ between an electron and an ion with charge Ze in electron-ion quantum plasmas can be represented by

$$V_\mathrm{MDH}(r) = -\frac{Ze^2}{r}\exp\left(-\frac{k_\mathrm{q} r}{\sqrt{2}}\right)\cos\left(\frac{k_\mathrm{q} r}{\sqrt{2}}\right), \tag{6}$$

where k_q $[\equiv (4m^2\omega_{\mathrm{p}e}^2/\hbar^2)^{1/2}]$ is the quantum wave number, m is the electron mass, $\omega_{\mathrm{p}e}$ $[= (4\pi ne^2/m)^{1/2}]$ is the electron plasma frequency, and n is the electron density. In these quantum plasmas, the Fermi electron temperature T_Fe is known to be $k_\mathrm{B}T_\mathrm{Fe} \ll (\hbar^2/4m)(\nabla^2 n_1/n_1)$, where k_B is the Boltzmann constant and n_1 is the electron density perturbation [8]. The dielectric function of these quantum plasmas becomes $\varepsilon_\mathrm{qp} = 1 + k_\mathrm{q}^4/k^4$ because of the quantum force term $(\hbar^2/4mn_0)\nabla(\nabla^2 n_1)$, due to the quantum Bohm potential [12, 13]. In a conventional classical plasma related to the standard Debye-Hückel potential, the dielectric function is given by $\varepsilon_\mathrm{cl} = 1 + k_\mathrm{D}^2/k^2$, where k_D is the Debye wave number. As it is seen, that this modified Debye-Hückel interaction potential [see (6)] is quite different from the standard Debye-Hückel screening [3] due to the oscillating part. The oscillatory behaviour of the effective potential is caused by the singular points of the function $(k^4 + k_\mathrm{q}^4)^{-1}$, i. e., the poles at $k^2 = \pm\mathrm{i}k_\mathrm{q}^2$, due to the contribution of the quantum Bohm potential term. It is shown that the Fourier transformation of $(k^2 \pm \mathrm{i}k_\mathrm{q}^2)^{-1}$ contains the oscillating exponential terms [12]. In a recent excellent work of Brodin et al. [14], a detailed discussion on the parameter space for various quantum plasma effects is given. It is found that the density and temperature range of quantum plasmas [14] including the Bohm potential effect are known to be around $10^{12} - 10^{21}$ cm^{-3} and $10^0 - 10^4$ K, respectively. Using the modified Debye-Hückel potential $V_\mathrm{MDH}(r)$ in the cylindrical coordinate system, the scattering phase shift can be obtained

$$\begin{aligned}\xi(k,b) = {}& \frac{m}{\hbar^2 k}Ze^2\\ &\cdot\int_{-\infty}^{\infty}\frac{\mathrm{d}z}{\sqrt{z^2+b^2}}\exp\left(-k_\mathrm{q}\sqrt{z^2+b^2}/\sqrt{2}\right)\\ &\cdot\cos\left(k_\mathrm{q}\sqrt{z^2+b^2}/\sqrt{2}\right),\end{aligned} \tag{7}$$

where the radial distance r between the electron and ion is replaced by $r = \sqrt{z^2+b^2}$. After some mathematical manipulations with a change of the variable such as $z = b\sqrt{t^2-1}$, the scattering phase shift is found to be

$$\begin{aligned}\xi(k,b) &= \frac{m}{\hbar^2 k}2Ze^2\mathrm{Re}\\ &\quad\cdot\int_1^\infty\frac{\mathrm{d}t}{\sqrt{t^2-1}}\exp\left(-k'_\mathrm{q}bt/\sqrt{2}\right)\\ &= \frac{2}{ka_Z}\mathrm{Re}K_0\left(k'_\mathrm{q}b/\sqrt{2}\right),\end{aligned} \tag{8}$$

since the integral expression of the second kind modified Bessel function K_ν of order ν [10] is given by $K_\nu(y) = [\pi^{1/2}/(\nu-1/2)!](y/2)^\nu \int_1^\infty \mathrm{d}p\mathrm{e}^{-yp}(p^2-1)^{\nu-1/2}$ for $\nu > -1/2$, where Re stands for the real value, $k'_\mathrm{q} \equiv k_\mathrm{q}(1-\mathrm{i})$ is the complex quantum wave number, i is the pure imaginary number, a_Z $(= a_0/Z)$ is the Bohr radius of the hydrogen ion with nuclear charge Ze, and a_0 $(= \hbar^2/me^2)$ is the Bohr radius of the hydrogen atom. If we use the standard Debye-Hückel potential $V_\mathrm{DH}(r)$ $[= (-Ze^2/r)\exp(-k_\mathrm{q}r/\sqrt{2})]$ in quantum plasmas without including the oscillating factor, the scattering phase shift $\xi'(k,b)$ becomes

$$\xi'(k,b) = \frac{2}{ka_Z}K_0(k_\mathrm{q}b/\sqrt{2}). \tag{9}$$

The total cross section $\sigma_\mathrm{el}(\bar{E})$ in units of πa_Z^2 for the elastic electron-ion collision including the oscillatory screening effects in quantum plasmas is then found to be

$$\frac{\sigma_\mathrm{el}(\bar{E})}{\pi a_Z^2} = 4\int_0^\infty \mathrm{d}\bar{b}\,\bar{b}\left\{1-\cos\left[\frac{2}{\sqrt{\bar{E}}}\mathrm{Re}K_0\left(\frac{\bar{k}'_\mathrm{q}\bar{b}}{\sqrt{2}}\right)\right]\right\}, \tag{10}$$

where $\bar{b}$ $(\equiv b/a_Z)$ is the scaled impact parameter, $\bar{E}$ $(\equiv E/Z^2Ry)$ is the scaled collision energy, Ry $(= me^4/2\hbar^2 \approx 13.6$ eV) is the Rydberg constant, and $\bar{k}'_\mathrm{q}(\equiv k'_\mathrm{q}a_Z) = \bar{k}_\mathrm{q}(1-\mathrm{i})$. In addition, the differential total cross section $\mathrm{d}\sigma_\mathrm{el}/\mathrm{d}\bar{b}$ is given by

$$\frac{\mathrm{d}\sigma_\mathrm{el}(\bar{E})}{\pi a_Z^2\mathrm{d}\bar{b}} = 4\bar{b}\left\{1-\cos\left[\frac{2}{\sqrt{\bar{E}}}\mathrm{Re}K_0\left(\frac{\bar{k}'_\mathrm{q}\bar{b}}{\sqrt{2}}\right)\right]\right\}. \tag{11}$$

However, if oscillatory screenings are absent in quantum plasmas, the total σ'_el and differential $\mathrm{d}\sigma'_\mathrm{el}/\mathrm{d}\bar{b}$ elastic electron-ion collision cross sections are, repectively, represented by

$$\frac{\sigma'_\mathrm{el}(\bar{E})}{\pi a_Z^2} = 4\int_0^\infty \mathrm{d}\bar{b}\,\bar{b}\left\{1-\cos\left[\frac{2}{\sqrt{\bar{E}}}K_0\left(\frac{\bar{k}_\mathrm{q}\bar{b}}{\sqrt{2}}\right)\right]\right\}, \tag{12}$$

$$\frac{\mathrm{d}\sigma'_\mathrm{el}(\bar{E})}{\pi a_Z^2\mathrm{d}\bar{b}} = 4\bar{b}\left\{1-\cos\left[\frac{2}{\sqrt{\bar{E}}}K_0\left(\frac{\bar{k}_\mathrm{q}\bar{b}}{\sqrt{2}}\right)\right]\right\}. \tag{13}$$

In order to explicitly investigate the oscillatory screening effects on the elastic collision process in dense electron-ion quantum plasmas, we set $\bar{E} > 1$ since the eikonal formalism is known to be valid for $\upsilon > Z\alpha c$ [7], where α $(= e^2/\hbar c \cong 1/137)$ is the fine structure constant and c is the velocity of light. Figure 1 represents the comparison between the scattering phase shifts ζ obtained by the modified Debye-Hückel screening and by the standard Debye-Hückel screening as a function of the scaled impact parameter $\bar{b}$. As shown, the phase shift obtained by the standard Debye-Hückel screening monotonically decreases with an increase of the impact parameter. However, the phase shift obtained by the modified Debye-Hückel screening shows a minimum position which cannot be found in the case of the standard Debye-Hückel screening. Figure 2 shows the phase shift ξ obtained by the modified Debye-Hückel screening as a function of the scaled impact parameter $\bar{b}$ for various

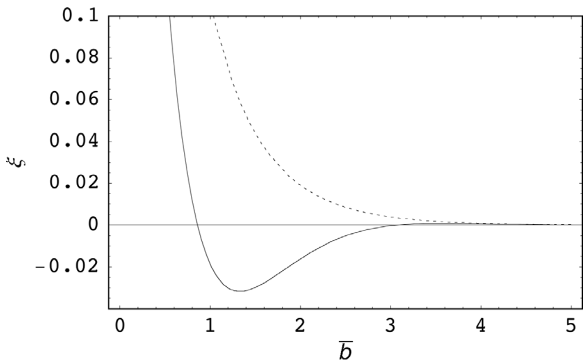

Fig. 1. The eikonal scattering phase shift ξ of the elastic electron-ion collision in quantum plasmas as a function of the scaled impact parameter $\bar{b}$ for $\bar{E} = 20$ and $\bar{k}_\mathrm{q} = 2$. The solid line represents the phase shift ξ obtained by the modified Debye-Hückel potential. The dotted line represents the phase shift ξ' obtained by the standard Debye-Hückel potential.

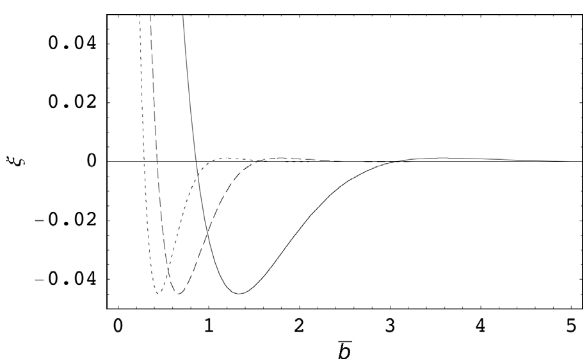

Fig. 2. The eikonal scattering phase shift ξ obtained by the modified Debye-Hückel potential as a function of the scaled impact parameter $\bar{b}$ for $\bar{E} = 20$. The solid line is the result for $\bar{k}_\mathrm{q} = 2$, the dashed line for $\bar{k}_\mathrm{q} = 4$, and the dotted line for $\bar{k}_\mathrm{q} = 6$.

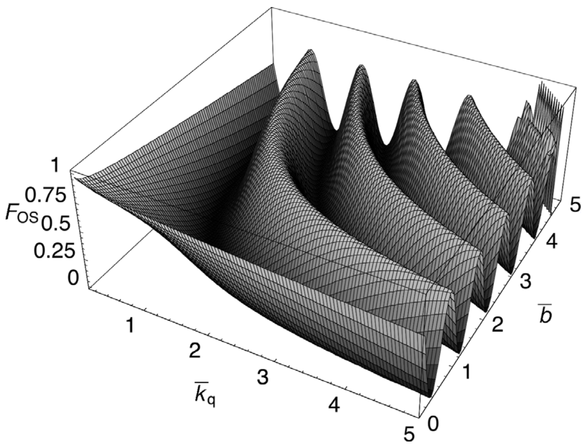

Fig. 3. The surface plot of the oscillatory screening effect F_{OS} on the differential cross section as a function of the scaled quantum wave number $\bar{k}_q$ and scaled impact parameter $\bar{b}$ for $\bar{E} = 20$.

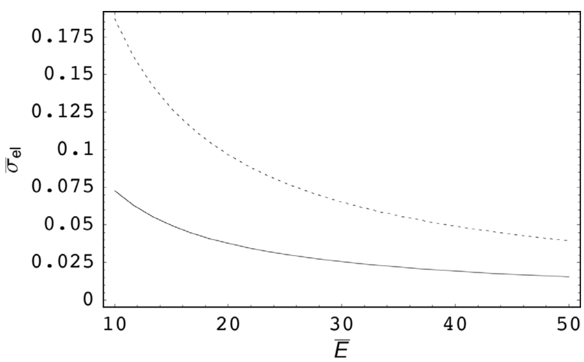

Fig. 4. The total scaled cross section $\bar{\sigma}_{el}$ of the elastic electron-ion collision in quantum plasmas as a function of the scaled collision energy $\bar{E}$ for $\bar{k}_q = 2$. The solid line represents the total scaled cross section $\bar{\sigma}_{el}$ obtained by the modified Debye-Hückel potential. The dotted line represents the total cross section $\bar{\sigma}'_{el}$ obtained by the standard Debye-Hückel potential.

values of the quantum wave number $\bar{k}_q$. It is shown that the minimum position of the scattering phase shift has receded from the target ion with decreasing the quantum wave number. Figure 3 represents the oscillatory screening effect F_{OS} $[\equiv (\mathrm{d}\sigma_{el}/\mathrm{d}\bar{b})/(\mathrm{d}\sigma'_{el}/\mathrm{d}\bar{b})]$ on the differential collision cross section as a function of the scaled quantum wave number $\bar{k}_q$ and the scaled impact parameter $\bar{b}$. As it is seen, the oscillatory screening effect sinusoidally decreases with an increase of the quantum wave number and impact parameter. Figure 4 shows the comparison between the total scaled cross section $\bar{\sigma}_{el}$ in units of πa_Z^2 obtained by the modified Debye-Hückel potential and by

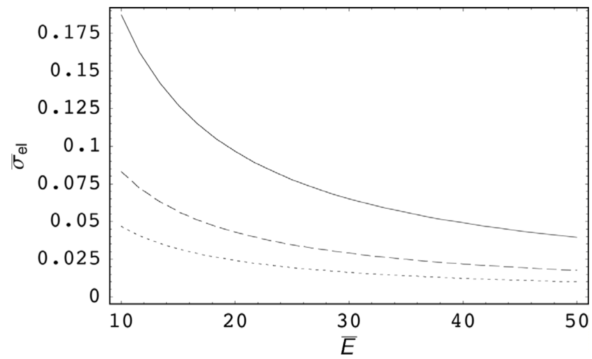

Fig. 5. The total scaled cross section $\bar{\sigma}_{el}$ obtained by the modified Debye-Hückel potential as a function of the scaled collision energy $\bar{E}$. The solid line is the result for $\bar{k}_q = 2$, the dashed line for $\bar{k}_q = 3$, and the dotted line for $\bar{k}_q = 4$.

the standard Debye-Hückel potential as a function of the collision energy $\bar{E}$. It is shown that the oscillatory screening effect suppresses the total cross section and also decreases with an increase of the collision energy. In addition, Fig. 5 represents the total scaled cross section $\bar{\sigma}_{el}$ $(\equiv \mathrm{d}\sigma_{el}/\pi a_z^2)$ obtained by the modified Debye-Hückel screening as a function of the scaled collision energy $\bar{E}$ for various values of the quantum wave number $\bar{k}_q$. It is also shown that the total cross section decreases with an increase of the quantum wave number. Hence, we find that an increase of the plasma frequency of the quantum plasma suppresses the total elastic electron-ion collision cross section.

In this work, we have found that the oscillatory screening behaviour caused by the quantum Bohm effect plays an important role in collision processes in quantum plasmas. Hence, the use of the accurate interaction potential is essential for the evaluation of the precise cross sections for various collision and radiation processes in plasmas. These results would provide useful information on the quantum screening effects of various atomic collision processes in dense quantum plasmas.

Acknowledgements

Y.-D. Jung gratefully acknowledges Director-General Professor O. Motojima and Professor D. Kato for warm hospitality and support while visiting the National Institute for Fusion Science (NIFS), Japan, as a long-term visiting professor. He also thanks Hanyang University, South Korea for granting him a year-long sabbatical leave.

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